Lesson 7-5: Areas of Regular Polygons

Remember these things?

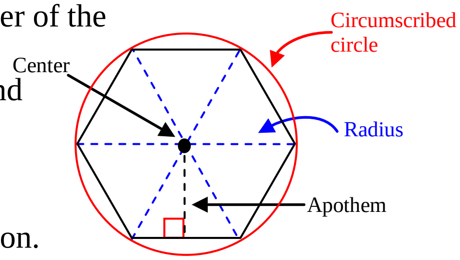
It has been awhile but do you remember regular polygons? These are polygons that have equal length sides. A couple examples are equilateral triangles, and squares. These polygons have special names based on the number of sides. If you need a refresher, review the list at the top of page 144 in the text. As a quick reminder, a pentagon is 5-sided and a hexagon is 6-sided.

Remember how to determine the total interior angle measure sum? It is $(n - 2)180$ where n is the number of sides.

Parts of a regular polygon

There are three new terms that refer to parts of regular polygons. These terms make a bit more sense if you realize that you can look at a regular polygon as an approximation of a circle. In fact, in 3D computer graphics, this is exactly how a circle is often represented. The more sides the regular polygon has, the closer it gets to "being" a circle. If this doesn't make sense, look at the figure below of a hexagon inside a circle. Picture an octagon inside that circle. The gaps between the octagon and the circle would be smaller. Now picture a 20-gon inside that circle. The gaps would be even smaller. The more sides, the closer the regular polygon approximates a circle.

Mathematically we will say that you can **circumscribe a circle** around any regular polygon. This would be the circle that touches every vertex of the polygon. Considering the following figure, you can see that you can draw a radius of the **circumscribed circle** at each vertex of the polygon. You can then see that the center of the circumscribed circle is the **"center" of the regular polygon**. This allows us to say that the regular polygon has a **center** and **radius** (distance from the center to any vertex).



The **apothem** of the regular polygon is the perpendicular distance (or segment) from the center to the side of the polygon.

The area of a regular polygon

For any regular polygon, two consecutive radii with the enclosed side form an isosceles triangle. The height of this triangle is the apothem and the base is the side of the polygon. Take a moment and examine the above figure so you can see this.

The regular polygon in the figure above is a hexagon. How many triangles do the radii of the hexagon form? It is pretty easy to see that there are six isosceles triangles formed by the radii. Could you use this to find the area of the hexagon? Sure! The area of each triangle is $\frac{1}{2}bh$ where b is the length of any side and h is the length of the apothem. Thus for an n-gon with side length s and apothem length a we have:

$$A = n \bullet \frac{1}{2} \bullet s \bullet a = \frac{1}{2}a(n \bullet s) = \frac{1}{2}ap$$ where p is the perimeter of the polygon.

Theorem 7-12

The area of a regular polygon is half the product of the apothem and the perimeter.

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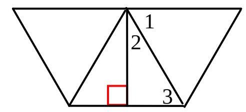
Examples

1. A portion of a regular hexagon has apothem and radii drawn. Find the measure of each numbered angle.



$\angle 1$ is a center angle of the polygon formed by 2 radii. There are six of these center angles (it is a hexagon). Since they go all the way around the center of the hexagon, their measures total 360. Thus $m\angle 1 = 360/6 = 60$.

$\angle 2$ is half a center angle (formed by the apothem which bisects the center angle) so $m\angle 2 = 60/2 = 30$.

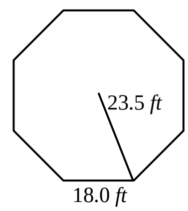
$\angle 3$ is the remaining angle of the triangle formed by the apothem and radius. The other two angles are 30 and 90, so we have a 30-60-90. $m\angle 3 = 60$.

2. Find the area of a regular polygon with twenty 12-in. sides and a 37.9-in. apothem.

Here n = 20, s = 12 and a = 37.9. Thus the perimeter $p = 20 \bullet 12 = 240$ and

$A = \frac{1}{2}ap = \frac{1}{2} \bullet 37.9 \bullet 240 = 4548\, in^2$.

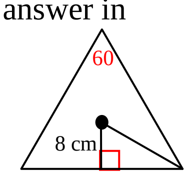
3. A library is a regular octagon. Each side is 18.0 *ft*. The radius of the octagon is 23.5 *ft*. Find the area of the library to the nearest 10 *ft*.



First we need to find the apothem. It is the side of a triangle whose hypotenuse is 23.5 and base is 9 (½ the side). Using the Pythagorean Theorem we find that

$$a^2 + 9^2 = 23.5^2;\ a = \sqrt{23.5^2 - 9^2} = \sqrt{471.25} \approx 21.7$$
$$Perimeter\ p = 8 \bullet 18 = 144$$
$$A = \frac{1}{2}ap = \frac{1}{2} \bullet 21.7 \bullet 144 = 1562.4 \approx 1560\, ft\ (rounding\ to\ nearest\ 10\ ft)$$

4. Find the area of an equilateral triangle with apothem 8 *cm*. Leave your answer in simplest radical form.



The radius bisects its corner angle. The corner angle measure is 60 since it is an equilateral triangle. Thus the measure of the angle formed by the radius and base/side is 30. Since the apothem forms a right angle with the base/side, we have a 30-60-90 triangle. The apothem is opposite the 30 degree angle so it is the shortest side. This makes the radius the hypotenuse (length 2·8=16). The length of the remaining leg of the triangle is $8\sqrt{3}$. Thus a side of the triangle is $2 \bullet 8\sqrt{3} = 16\sqrt{3}$ and the perimeter is $3 \bullet 16\sqrt{3} = 48\sqrt{3}$. The area $A = \frac{1}{2}ap = \frac{1}{2} \bullet 8 \bullet 48\sqrt{3} = 192\sqrt{3}\, cm^2$.

Homework Assignment

p. 382 #1-21 odd, 25, 27-31, 33, 35-40